Peak Analysis of Grayscale Image: Algorithm and Application

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Abstract

Considered as topographic reliefs, grayscale images can be decomposed into a number of peaks that can be stored in the data structure of a tree. This decomposition is called the peak analysis of grayscale images in this paper. Its mathematical definition and its fast algorithm based on the watershed transform are both presented. Due to its intuitive definition, there is a wide range of applications for the peak analysis. One of them was to measure the quantum dots in the AFM photos. The segment result proves to be very accurate and insensitive to noise to some degree.

Keyword: Peak Analysis; Grayscale Image; AFM photo.

I. Introduction

By using mathematical morphology methods, grayscale images can be decomposed into a tree structure. The tree representation of the original image can greatly facilitate the analysis of the image, including the aspects of image filter, image segment, pattern recognition and so on.

Reference [2] constructs a binary component tree (BCT) whose nodes correspond to connected components of the level sets of an image, linked by inclusion between successive levels. Then a filter is defined to decide if a node on the tree should be removed or preserved, according to the so-called attribute signature extracted from the branch containing the node. In virtue of the new filter, a new method of image segment is presented.

Reference [3] first presents the notion of the level-k connectivity of a grayscale image and thus the level-k connected components of a gray level image. By iteratively extracting the connected components of the grayscale image with increasing value of k, a tree of connected components can be constructed. The authors refer this structure as grayscale component tree (GCT).

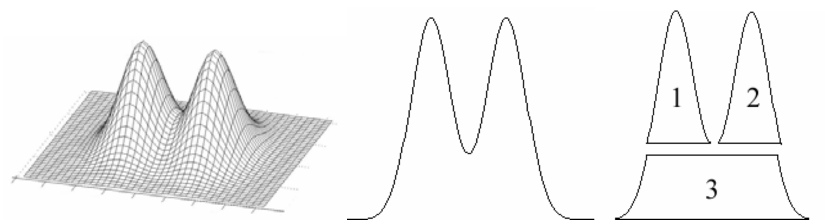
The two preceding trees are different in the content of their nodes, one with binary components whereas the other with gray level components. However, they are isomorphic [3]. Notice that the so-called GCT is actually a kind of binary decomposition. Therefore the depth of both trees generally equals the image gray level (e.g., 256).

In this paper, we propose a new notion of grayscale image decomposition. We use the peaks (or connected gray level images) as the basis of the deposition, and link the peaks to form a tree according to the inclusion of the nonzero domain of the peaks. The constructed tree is essentially a gray level decomposition of the image, completely distinct from the BCT and GCT. Therefore the constructed tree is not isomorphic with the already-existing trees and the depth of the tree is much smaller (typically 6-7).

The paper is organized as follows. The basic concepts of the peak, peak analysis and peak tree construction will be detailed in Section II. In Section III, the fast algorithm of peak analysis based on Watershed Transform is proposed. In Section IV, we discuss the application of peak analysis on the measure of the quantum dots on atomic force microscopy (AFM) photos, especially focusing on some problems during the post process of each peak and in the search of the tree.

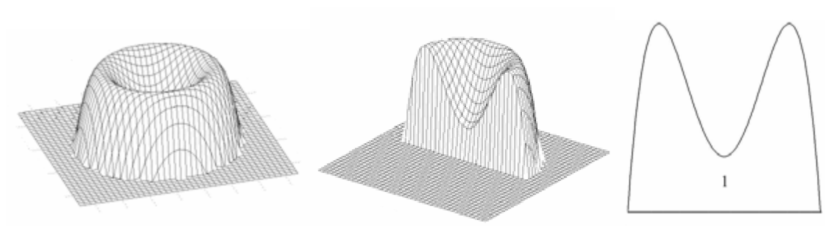
II. Peak Analysis

A. Definition of Peak



（a） （b） （c）

Fig 1 Grayscale image containing three peaks



（a） （b） （c）

Fig 2 Volcano peak

First we need the following basic definitions：

1) Grayscale image *f*： $Z^2 \rightarrow \Re$， where Z is the set of integers while R the set of real numbers. And let *FUN* denote the set of all the grayscale images.
2) The binary adjacency relative $\Gamma \subset \left\{(Z^2, Z^2)\right\}$ of the elements on the definition domain of *f*, namely if two points $x1, x2$ are adjacent $\Leftrightarrow$ $(x1, x2) \in \Gamma$ 。 In addition if two region E_1, E_2 are adjacent $\Leftrightarrow$ $\exists x1 \in E_1, x2 \in E2$, *x*1,*x*2 are adjacent。
3) Level set of grayscale image *f* at the height of $h \in \Re$: $LS_h(f) = \left\{x \in Z^2 \mid f(x) \geq h\right\}$
4) Region $E \subset Z^2$ is connected $\Leftrightarrow$ $\forall x \in E, \forall y \in E$, $\exists$a path $p \subset E$, *p* connects *x* and *y*.
5) Connected components. $\forall X \subset Z^2 \; \exists Y_1, Y_2, Y_3, ... \Rightarrow X = \cup Y_i$ where $\forall i, Y_i$ is a connected region, and $\forall j \neq i$, Y_i and Y_j are not adjacent. Y_i are the connected components of *X*.
6) Reconstruction. $\forall E \subset X$, the reconstruction of *X* from *E* is $REC_X(E) = \cup Y_k$, where Y_k is

the connected component of *X*, and $Y_k \cap E \neq \Phi$

7) Grayscale image $f \in FUN$ is a peak $\Leftrightarrow$ $\forall t_1, t_2 \in \Re, t_1 < t_2 \rightarrow$ $LS_{t1}(f), LS_{t2}(f)$ are both connected regions, and $LS_{t1}(f) \supseteq LS_{t2}(f)$

.

Fig 1 and Fig 2 show two examples of peaks. In Fig 1, (a) is the 3D view of the grayscale image, (b) is its profile. From (c) we can find that after the peak 1 and 2 were cut off from the original image, the residual plateau was also a peak. Therefore this grayscale image contains three peaks in all. Fig 2 illustrates a volcano, which is just a single peak according to the definition of the peak. See Fig 2(c).

B. Peak Analysis of Grayscale Images

First we need the following definitions:

1) Regional maximum. Connected region E is the regional maximum of grayscale image *f* $\Leftrightarrow$ ① $\exists h, \forall x \in E, f(x) = h$; ② $\forall x \in Z^2, x \notin E, \exists y \in E$, and *x,y* are adjacent $\rightarrow f(x) < f(y)$

2) Top hat peak. Let *RM(f)* denote the set of all the *regional maximum* of *f*. $\forall E_i \in \mathrm{RM}(f)$, then we can derive that:

$$\exists h_{\min} = \inf \left\{ h \mid \forall E_j \in RM(f), j \neq i, \rightarrow E_j \notin REC_{LS_h(f)}(Ei) \right\}, \qquad \textbf{(1)}$$

Let grayscale image f_i be defined as:

$$f_i(x) = \begin{cases} f(x) - h_{\min} & x \in REC_{LS_h(f)}(Ei) \\ 0 & \text{others} \end{cases}$$

It can be easily seen that f_i is a peak. Therefore we call f_i the top hat peak of *f* below the *regional maximum Ei*. And we denote the set of all the top hat peaks of *f* as *TP(f)*.

3) Peak Analysis. *Peak Analysis* is defined as the process of the iterative extraction of *TP(f)* from *f* until it contains no top hat peaks. Following is the description of the process:

```
Peak Analysis(f)
  Let t=1;  f^(1) = f ;
  While f^(t) ≠ CONSTANT
    Extract TP(f^(t)),
    rewrite it as TP^(t)(f) for simplicity.
    f^(t+1) = f^(t) − Σ_i f_i^(t) , where f_i^(t) ∈ TP^(t)(f)
    t = t + 1;
  end while;
end.
```

Finally we denote $PA(f) = \cup TP^{(t)}(f)$, which is right the *Peak Analysis* of *f*.

C.Tree Representation of Peak Analysis

First we need the following definition:

1) The non-zero domain of grayscale image *f*: $NZD(f) = \{x \in Z^2 \mid f(x) > 0\}$

$\forall$ Grayscale image *f*, $PA(f)$ its Peak Analysis, which has the following properties:

1) $f = \sum_t \sum_i f_i^{(t)}$, where $f_i^{(t)} \in TP^{(t)}(f)$,

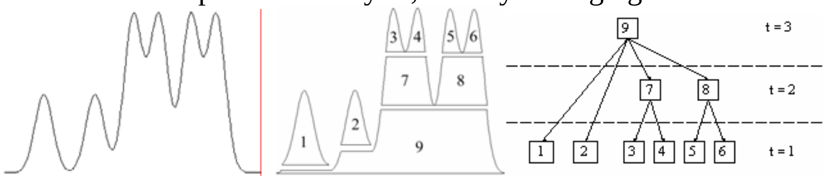
2) $\forall t, \forall i, j, i \neq j \rightarrow NZD(f_i^{(t)}) \cap NZD(f_j^{(t)}) = \Phi$,

3) $\forall t_1, t_2, t_1 < t_2, \forall i, j, NZD(f_i^{(t_1)}) \cap NZD(f_j^{(t_2)}) \neq \Phi \rightarrow$
$NZD(f_i^{(t_1)}) \subset NZD(f_j^{(t_2)})$,

Therefore the *peak analysis* of a grayscale image can be naturally represented by a tree structure, on which whether Node A is the parent of Node B depends on whether *NZD(A)* contains *NZD(B)* immediately. Here Containing immediately means if $NZD(f_i^{(t_1)}) \subset NZD(f_j^{(t_2)})$, then

$\sim(\exists t_3, k,\ NZD(f_i^{(t_1)}) \subset NZD(f_k^{(t_3)}) \subset NZD(f_j^{(t_2)})$, where $t_1 < t_3 < t_2$).

Fig 3 is an example of *Peak Analysis*. For the sake of simplicity, the grayscale image is shown as a 1D function. Fig 3(a) is the original pattern, and Fig 3(b) shows the decomposed peaks of 3(a). And the tree constructed from the *Peak Analysis* is shown in Fig 3(c), where nodes on the same horizontal line *t* are extracted at the same step *t* of the analysis, namely belonging to $TP^{(t)}$.



(a) (b) (c)

Fig 3 An example of *Peak Analysis*

III. Algorithm to Extract the Top Hat Peak

A. Watershed Transform

Watershed Transform was first proposed in 1979[4] and soon becomes a widely used technology in image segment [5]. Its basic notion derives from the concepts of *Catchment basins* and *watershed* from topography. The most famous example is the *great divide* in America [1]. When a raindrop falls on the east of the *divide*, it will finally flow to the Atlantic; on the west, to the Pacific. Taken a grayscale image as a topographic relief, Watershed Transform converges the positions from where a raindrop will flow to the same valley into the same region (*Catchment basins*), while the lines where *Catchment basins* touch each other are called the watershed. See Fig 4 from [1].

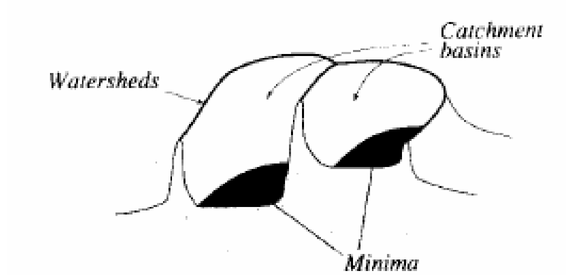


Fig 4 Minimum, Watershed and Catchment basin

Since our purpose is to segment the peak, we need to apply watershed transform to the grayscale image *–f*, resulting in the *Catchment basins* {CB_1, CB_2, ….} and the region of the watershed *WS*. In addition, Let's denote

$$\partial CB_i = \{x \in WS \mid x \text{ is adjacent to } CB_i\},$$

namely the watershed surrounding the CB_i.

Therefore we can reach the following conclusions:

1) $\forall CB_i$, there is only one *regional maximum* of *f* in CB_i, denoted as $RMAX_i$. That is to say, if limited in CB_i, *f* is a peak.

2) $\forall CB_i$, let $h_i = \max\{f(x) \mid x \in \partial CB_i\}$, $\forall h$,

$$\text{if } h \ge h_i, \text{ then } \forall RMAX_j, j \ne i, RMAX_j \notin REC_{LS_h(f)}(RMAX_i);$$

$$\text{if } h < h_i, \text{ then } \exists RMAX_j, j \ne i, RMAX_j \in REC_{LS_h(f)}(RMAX_i).$$

That is to say that h_i is the height at which the *top hat peak* corresponding to the regional maximum $RMAX_i$ should be cut off. Therefore we can extract this top hat peak according to the height h_i and obtain the following algorithm for *top hat peak extraction*.

B. Algorithm for Top Hat Peak Extraction

Let *f* denote the grayscale image. The algorithm for top hat peak extraction can be written as follows:

```
Hat Peak Extract (f)
  Apply Watershed Transform to the -f to attain {CB_i}
  and WS
  for all CB_i
    Search for the maximum height h_i on ∂CB_i
    Let top hat peak f_i=0
    for all x ∈ CB_i
      if f(x) >= h_i, let f_i(x) = f(x) - h_i; end if
    end for
  end for;
end process;
```

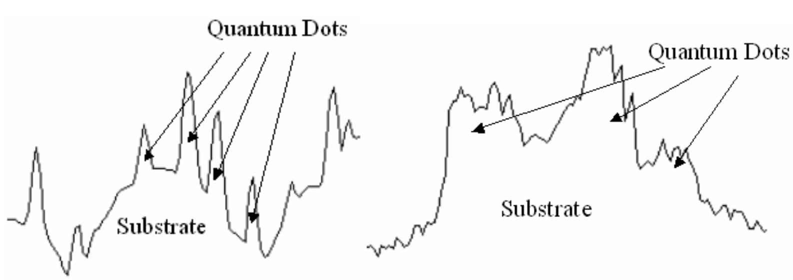
At last we get $TP(f) = \{f_i\}$;

IV. Application of Peak Analysis

A. Atomic Force Microscopy (AFM) photo

AFM photo is the observation of the growing result of micro materials. On the photo the intensity value of each pixel corresponds to the height of the growing material. The proposed application of the *Peak Analysis* is based on the photo of the quantum dots as well as quantum wells, which is characterized with following properties：

1) The substrate on which the quantum dots are grown is also fluctuating. Consequently a simple threshold is not appropriate for the segment purpose, See Fig 5(a). Another reason for the requirement of a more robust segment technology is the widely presence of the obvious noise on the photo, see Fig 5(b).



(a) (b)

Fig 5 two kinds of quantum dots

2) The grown quantum dots and quantum wells are of diverse shapes: such as circles, annuluses, crescents and even more irregular shapes, see Fig 8(a). However they are in common that they are all highlands that stand in a given height range and span a range of size. Although quantum dots may not necessarily be a strict *Peak* we defined above, they are sure to be one of the nodes (including the whole subtree of the node) on the *Peak Tree*. Therefore searching the *Peak Tree* is the exact way to extract the quantum dots.
3) AFM grayscale image is a two-variable function that has the characteristic of fractal. Sorting by the size, there exist the same pattern from substrate, quantum dots to noise, see Fig 5 (a) (b). Thus the size for qualified quantum dots are required to be defined in advance by the user.

Different from the problem of edge locating of Marr's theory. If we adopt the criterion of the maximum of the first differential, the quantum dots we segment will be smaller than they really are. See Fig 6, according to the maximum of first differential, we will get the part of A, greatly differing from the desirable result of B.

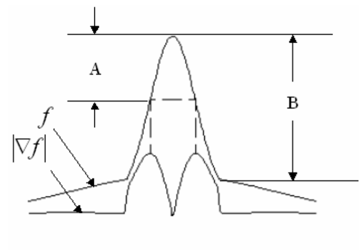
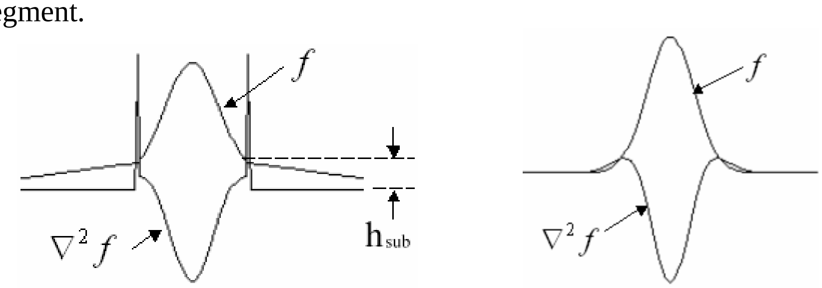


Fig 6 the segment error resulted from the maximum of $|\nabla f|$

B. Algorithm to search for the quantum dots

Suppose we have applied *peak analysis* to the grayscale image *f*, and saved the result in the peak tree *PT(f)*. Now in order to locate the quantum dots, the remaining job is to traverse the whole tree, visit each node (namely each peak) to first do some process to the peak (the following Step 1) and then to judge if it is a qualified QD according to its size, height and other attributes (Step 2 and 3).
Let's denote the node (peak) being visited as f_i.

1) Determining the existence of the substrate in the peak. If so, eliminate it.
   One feature of *peak analysis* is to take into account only the altering trend of the function value (the shape) of a peak while leave its altering speed (the slope) alone. Please refer to Fig 6, there exist two uncontinuous breaks of the gradient of function *f* and the slow-decreasing part outside the breaks of the peak *f* should be deemed as the substrate, which should be eliminated before the final segment.



(a) (b)

Fig 7 (a) Peak containing heterogeneous substrate (b) Peak without substrate

   Since a sudden drop of the first differential of function *f* corresponds to a large value of the second differential of *f*, which in multivariable function can be estimated by the Laplacian value ($\nabla^2 f$) of *f*, we can judge the existence of substrate by finding that in the lower part of the *peak f* there exist large abstract values of $\nabla^2 f$ compared with those in the higher part of *f*. Let h_{sub} denote the value (height) of *f* where the high Laplacian value appears. In this way, by removing the part of the peak that is lower than h_{sub}, we can eliminate the substrate. See Fig 7.

2) Denote $pf_i = \sum f_k$, where f_k is the node on the subtree of f_i (including f_i itself). From the view of topography, pf_i is whole part of the relief that stacks on the peak f_i. Let $h_i = \max(f_i)$, $ph_i = \max(pf_i)$, namely the height of peak f_i itself and the height of the total height of mountain above f_i separately.

3) Let $NZD(f_i)$ denote the nonzero domain of $f_{i,}$ extract the boundingbox of $NZD(f_i)$ as $BBox(NZD(f_i))$, record the widths of this rectangle as w_1,w_2。
4) Let user input the qualified height range of QD as [H_{min}, H_{max}], size range as [W_{min}, W_{max}], accompanied with the ph_i, w_1, w_2 to determine whether current peak is a qualified QD.

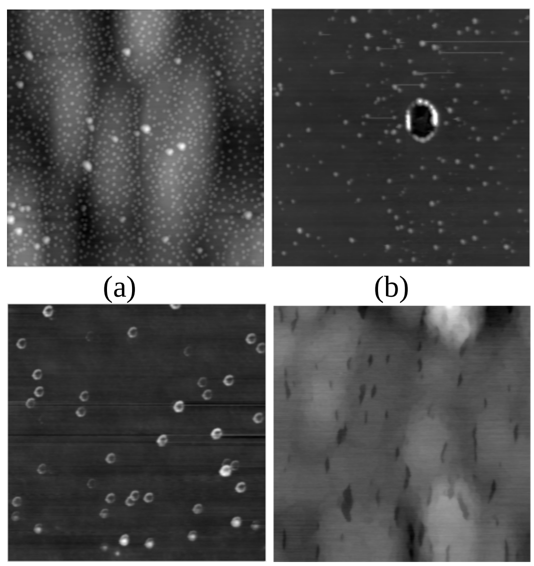
C. Experiment Results

The program code of this paper is written in Matlab, and utilizes its *watershed* function in its image processing toolbox. To extract the peaks of a 256 by 256 image, *watershed* generally need to be called 6~7 times, which takes most of the time of process. To extract the whole QDs, a PC with PIII costs about 4~5 seconds.

Fig 8, Fig 9 illustrate the segmenting results of some typical AFM photo. It can be clearly seen that the segment results are quite correct and are unsensitive to the existence of strong noise. In addition, a variety of shapes, such as annuluses ,circulars and rectangles, can all identified.

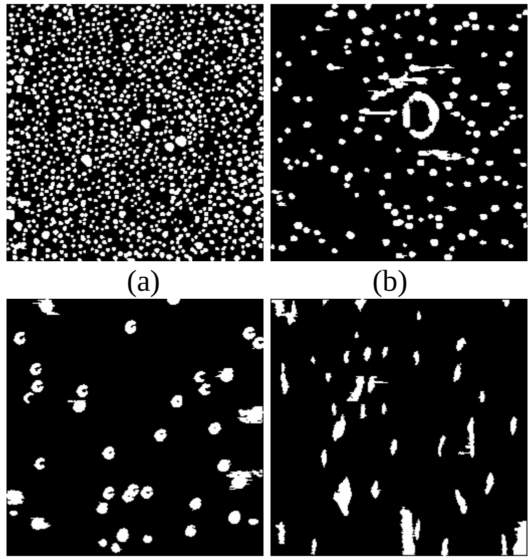
V. Conclusion

Due to its intuitive excellent properties and fast calculate, *Peak analysis* promises to be a powerful tool in image processing. Its applications of denoising, image filtering, and even image compression are our ongoing work.



(a) (b) (c) (d)

Fig 8 AFM photos. Notice (d) contains quantum wells



(a) (b) (c) (d)

Fig 9 Segment results

References

[1] Luc Vincent and Pierre Soille, "Watersheds in Digital Spaces: An Efficient Algorithm Based on Immersion Simulations," IEEE Transactions of Pattern Analysis and Machine Intelligence, vol. 13, no. 6, June 1991, pp. 583-598.

[2] R. Jones, “Connected filtering and segmentation using component trees,” Comput. Vis. Image Understanding, vol. 75, pp. 215–228, 1999.

[3] Braga-Neto, U.; Goutsias, J.; “Grayscale level connectivity: theory and applications” Image Processing, IEEE Transactions on , Volume: 13 , Issue: 12 , Dec. 2004 Pages:1567 – 1580

[4] S. Beucher and C. Lantuejoul, "Use of watershed in contour detection," in International Workshop on Image Processing: Real-time edge and motion detection /estimation. Rennes, France, 1979, pp. 17-21.

[5] S. Beucher and F. Meyer, "The morphological approach to segmentation: the watershed transformation," in Mathematical morphology in image processing, E. Dougherty, Ed. New York: Marcel Dekker, 1993, pp. 433-481.Baldonado, M., Chang, C.-C.K., Gravano, L., Paepcke, A.: The Stanford Digital Library Metadata Architecture. Int. J. Digit. Libr. 1 (1997) 108–121.